

Curzon's Calculation Policy

General Notes

- By the end of Year 6, most children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.
- Children should not be made to go onto the next stage if:
 - 1) they are not ready
 - 2) they are not confident
- **Book layout Years 1-6 is vital:**
 - One digit per square
 - KS2 to draw a margin

(Reception use white boards/ work sheets rather than books)

- All the way through the school children will use concrete materials and pictorial representations as appropriate.
- All children must have secure understanding of: number bonds, estimation, inverse, times tables, place value and partitioning.
- We say **hundreds (100), tens (10), units (1)**
- When teaching subtraction, use the word 'exchanging' not 'borrowing' or 'carrying'.
- In KS1 pupils are introduced all the time to new concepts that are revisited in lots of practical ways. This helps them develop the confidence to then have another go at another time.

Children in upper key stage 2/higher attainers who are ready to move on should be using the more advanced written methods.

Children should be encouraged to:

- approximate their answers before calculating (this is critical, so that they can then check the sensibility of their calculation)
- check their answers after calculation using an appropriate strategy
- to consider if a mental calculation would be appropriate before using written methods
- to record through a range of methods, including drawings

Curzon follows a mastery curriculum. Concepts will be taught using concrete, visual and abstract strategies. Children in all classes will work on deepening their learning through activities, such as: missing number questions, reasoning and problem questions, bar modelling and word problems.

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies:

Mental recall of number bonds to 10 and 100

$$6 + 4 = 10$$

$$25 + 75 = 100$$

$$\square + 3 = 10$$

$$19 + \square = 20$$

Use 'near doubles'

$$6 + 7 = \quad \text{is the same as double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (\text{adding the 10s: } 30 + 40) + (\text{then adding the units: } 4 + 5) = 79$$

This is often first taught using a number square until children become confident enough to do it in their heads.

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens i.e. 96, 106, 116, 126, 136 and then in ones ie 137, 138, 139, 140, 141, 142, 143)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds i.e. 360, 260, 160)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

'Inverse' calculations are:

$$55 - 19 = 36$$

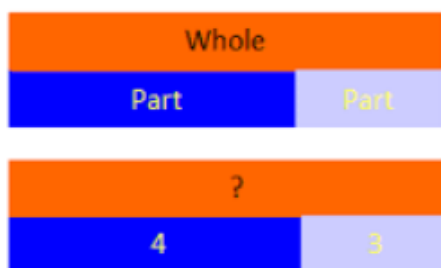
$$55 - 36 = 19$$

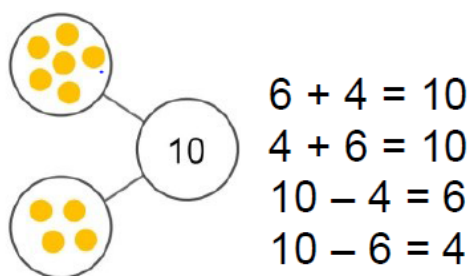
Use known number facts

$$3 + 4 = 7 \text{ so } 30 + 40 = 70$$

Language of 'whole' and 'part'

Children are taught that in addition the 'parts' are added together to make the 'whole'





*MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED.
THEY ARE NOT REPLACED BY WRITTEN METHODS.*

WRITTEN CALCULATION STRATEGIES FOR ADDITION

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

Stage 1 (approx Reception and Y1)

Reception

Counting activities i.e. counting spots on ladybirds, counting collections of objects, counting 1p coins in cafe.

Number rhymes/ action rhymes

Saying 1 more than with numbers up to 20.

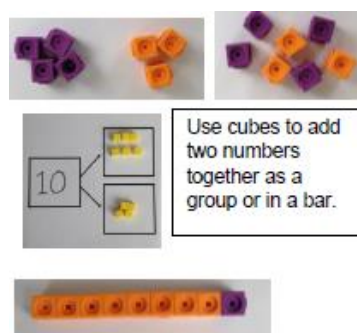
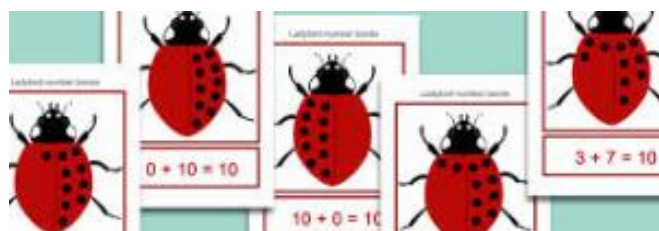
Jumping along number lines

Counting on 'one more' / 'one less' on washing lines

Start recording informally, using own symbols and pictures.

Children are provided with opportunities to use mathematical language within their play.

Adding 3 single digit numbers.



Initially use a number track to count on for addition, counting on from the largest number:



$$5 + 4 = 9$$

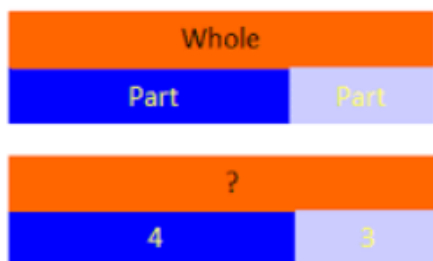
'Put your finger on number five. Count on (count forwards) four.'

Year 1

Counting and practical activities

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.

They develop ways of recording calculations using pictures including **bar models**.



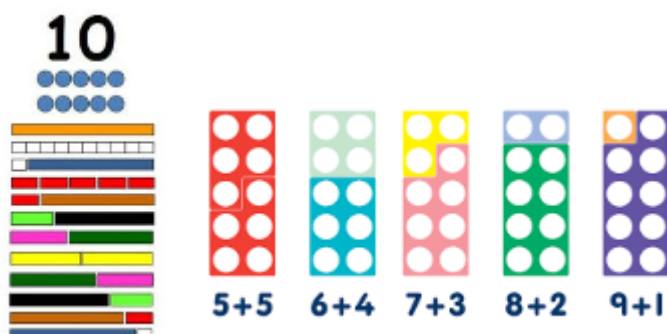
Find 'one more' than a number from 1 to 100.

Find '10 more' than a number from 1 to 100

Use vocabulary associated with addition (plus, add, sum of, total, altogether)

Begin to use the + and = sign to record mental calculations

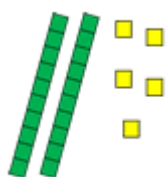
Begin to know by heart all number bonds to 10. Represented through Cuisenaire rods and Numicon.



Use knowledge to know that addition can be done in any order to do mental calculations more efficiently

Recall doubles and halves to 10 and 20.

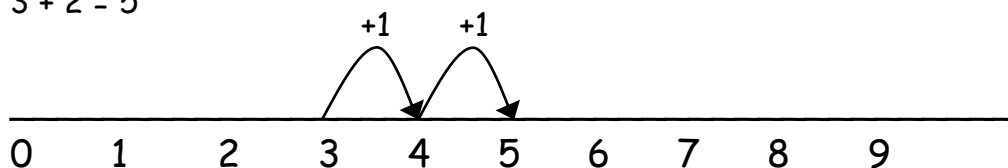
Begin to **partition 2 digit numbers** up to 100 using concrete apparatus, such as Dienes.



$$20 + 5 = 25$$

Use **number lines** and practical resources (such as Numicon) to support calculations with numbers up to 20 (teachers *demonstrate* the use of the number line).

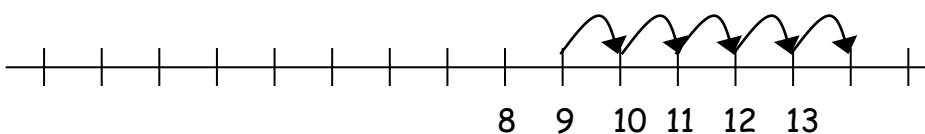
$$3 + 2 = 5$$



Children then begin to use numbered lines to support their own calculations to count on in ones.

$$8 + 5 = 13$$

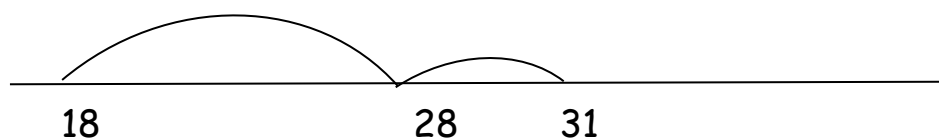
+1 +1 +1 +1 +1



Move onto blank number lines $18 + 13$

+10

+3



Missing numbers need to be placed in all possible places.

$$3 + 4 = \square \quad \square = 3 + 4$$

$$3 + \square = 7 \quad 7 = \square + 4$$

Add 1 digit and 2 digit numbers to 20 including 0.

Stage 2 (approx Y2)

Know by heart number bonds to 20. Derive and use related simple addition bonds to 100 (e.g. $60+40=100$) Move onto bonds $55+45$, then harder bonds $46+54$.

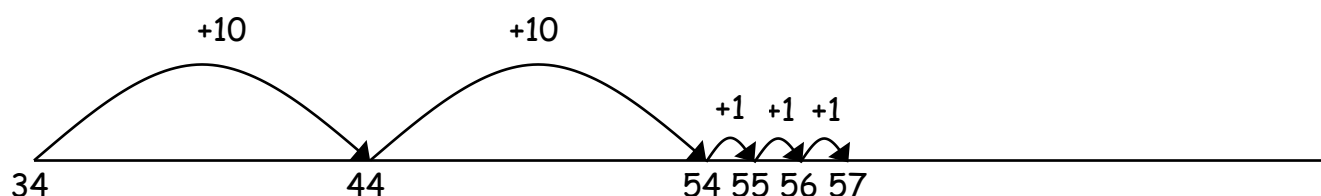
Missing number problems e.g. $14 + 5 = 10 + \square$ $32 + \square = 100$

$$35 = 1 + \square + 5$$

Children will use 'empty number lines' themselves starting with the larger number and counting on.

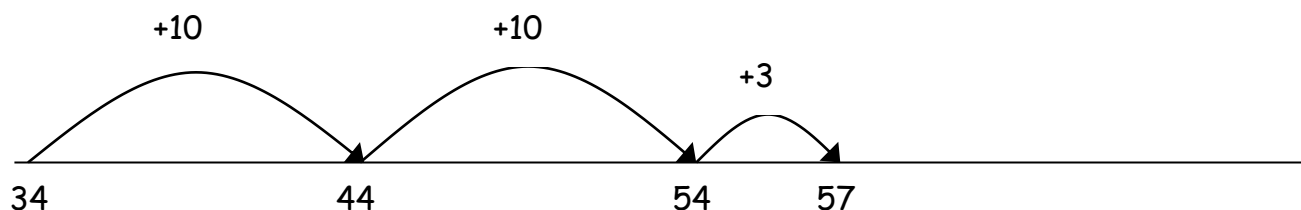
✓ First count on in tens then in ones.

$$34 + 23 = 57$$



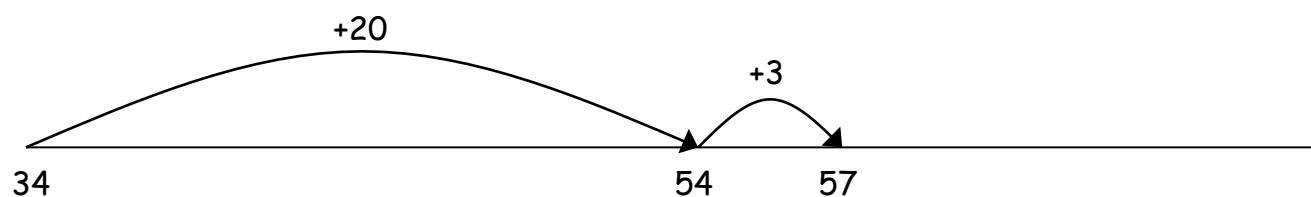
✓ Then help children to become more efficient by adding the ones in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



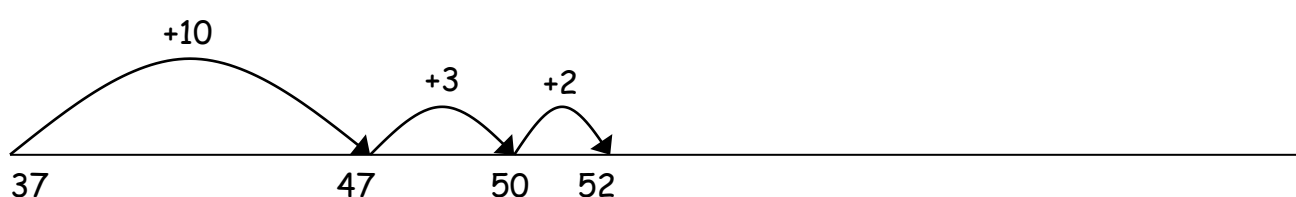
- ✓ Add the tens in one jump and the ones in one jump.

$$34 + 23 = 57$$



- ✓ Bridging through ten can help children become more efficient.

$$37 + 15 = 52$$



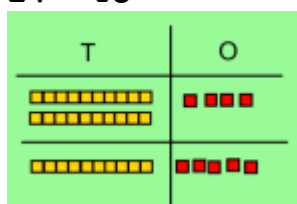
Add larger numbers:

- a two-digit number add units
- a two-digit number add tens
- two two-digit numbers
- three one-digit numbers

Children need to show that addition of two numbers can be done in any order (commutative)
They check their answers by adding numbers in different orders.

Children will start to add up units and then tens (no carrying at this stage)

$$24 + 15 =$$



After practically using the Dienes blocks, children can draw the blocks to help them to solve addition calculations. They will then record the calculation vertically imagining the Dienes blocks.

$$\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$$

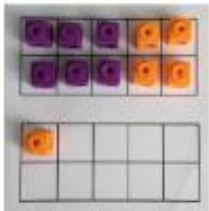
Stage 3 (approx Y3)

Use near doubles e.g. $15 + 14 =$
Partitioning with up to 3 digits

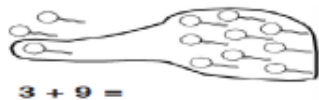
Children will continue to use empty number lines with increasingly large numbers, including compensation* where appropriate.

Regrouping-mental strategy

$$6 + 5 = 11$$

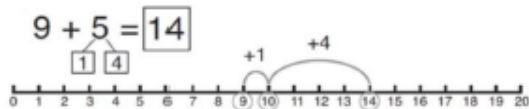


Start with the bigger number and use the smaller number to make 10.



$$3 + 9 =$$

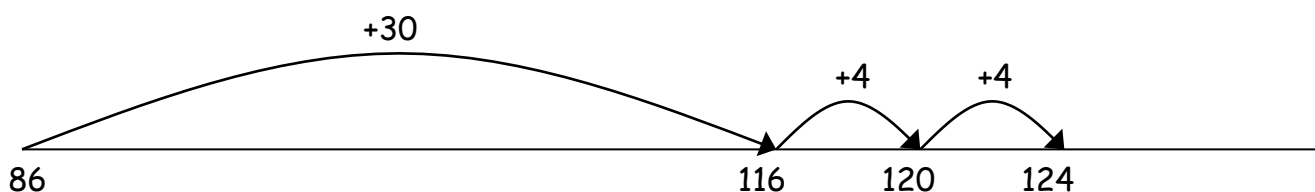
Use pictures or a number line. Regroup or partition the smaller number to make 10.



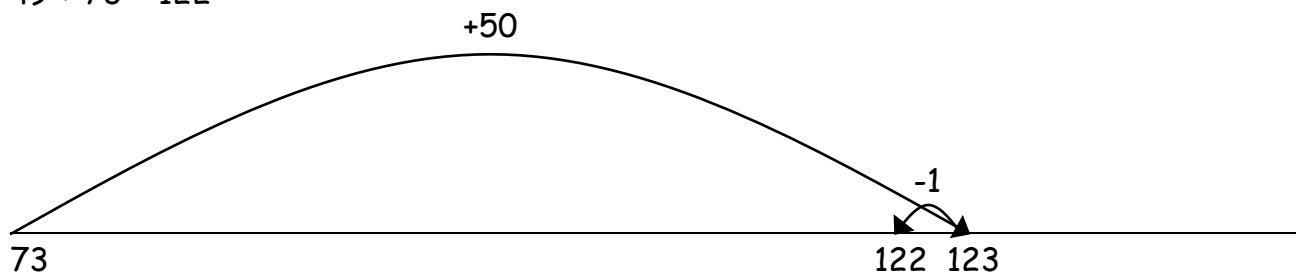
$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10.
How many more do I add on now?

✓ Count on from the largest number irrespective of the order of the calculation.
 $38 + 86 = 124$



✓ Compensation
 $49 + 73 = 122$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

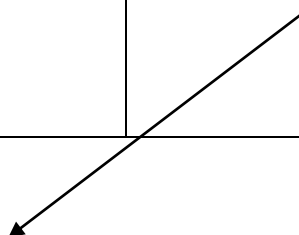
✓ From this, children will begin to carry below the line - column addition.
 Use Dienes and draw diagrams.

Hundreds	Tens	Units

$$\begin{array}{r} 29 \\ + 33 \\ \hline \end{array}$$

Add up the units and carry ten units into the tens column.

Hundreds	Tens	Units



$$\begin{array}{r} 29 \\ + 33 \\ \hline 62 \\ \hline 1 \end{array}$$

Move onto solving without Dienes and diagrams.

$$\begin{array}{r} 15 \\ + 18 \\ \hline 33 \\ \hline \cancel{1} \end{array}$$

Children are taught to cross through the carried digit once they have added it back in.

- ✓ Add larger numbers
- a three-digit number add units
- a three-digit number add tens
- a three-digit number add hundreds

Stage 4 (approx Y4)

Consolidate vertical informal methods.
Extend to numbers up to 4 digits.

Move onto “column addition”. Children should be able to make the choice of revisiting Dienes apparatus if experiencing any difficulty.

$$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ \hline \cancel{1} \end{array}$$

$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ \hline \cancel{1} \end{array}$$

$$\begin{array}{r} 2367 \\ + 85 \\ \hline 452 \\ \hline \cancel{11} \end{array}$$

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money (with or without adjustment from the pence to the pounds);*
- ✓ *Move onto numbers with up to 4 digits.*
- ✓ *Focus on applying in one step and then two step problems.*
- ✓ *Later in the year, know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

Stage 5 (approx Y5)

Children should extend the carrying method to numbers with at least four digits.

$$\begin{array}{r} 5871 \\ + 4751 \\ \hline 10622 \\ \cancel{11} \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \cancel{111} \end{array}$$

Using similar methods, children will:

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm;
- ✓ estimate first;
- ✓ focus on applying in multi- step word problems;
- ✓ Use rounding to estimate answers.

Stage 6 (approx Y6)

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ \cancel{111} \end{array}$$

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ \cancel{111} \end{array}$$

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \cancel{111} \end{array}$$

Using similar methods, children will

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more decimals with up to four digits and either one or two decimal places;
- ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71;
- ✓ different vocabulary for addition;
- ✓ use addition as part of multi-step problems;
- ✓ Use estimation to check answers.

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies:

Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

Find a small difference by counting up (e.g. 80, 81, 82)

$$82 - 79 = 3$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens i.e. 76, 66, 56, and then in ones i.e. 55, 54, 53, 52)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds i.e. 360, 260, 160)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

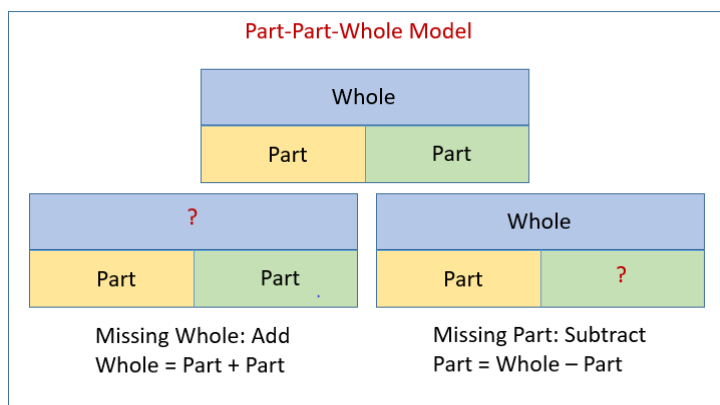
Inverse calculations are:

$$55 - 19 = 36$$

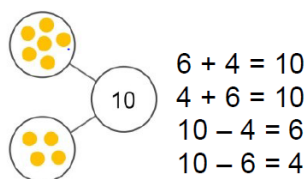
$$55 - 36 = 19$$

Language of 'whole' and 'part'

Children are taught that in subtraction, they take away a 'part' from the 'whole'



**MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED.
THEY ARE NOT REPLACED BY WRITTEN METHODS.**



WRITTEN METHODS FOR SUBTRACTION

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

Stage 1 (approx Reception and Y1)

Reception- mostly mental methods

Counting backwards

Saying '1 less than' with numbers up to 10.

Number rhymes (e.g. Five Currant Buns, Ten Green Bottles)

Finding 'the number before' on number lines

Starting on 8, jump back 5. What do I land on?

Giving change in 1ps in the class shop

Hiding some objects, How many are left?

Starting recording informally using own symbols and pictures.

Children are provided with opportunities to use mathematical language within their play.

Key questions:

How many more than..?

How many less than..?

What is the difference between...?

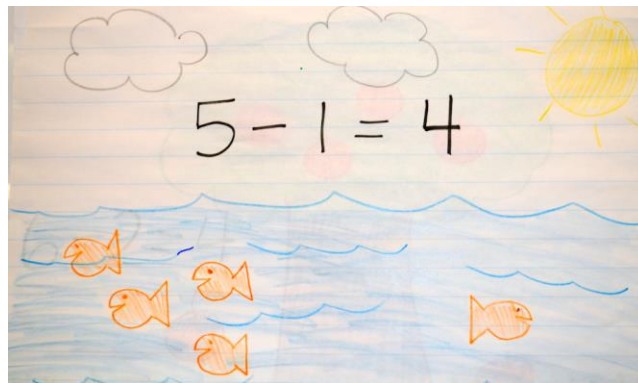


$$9 - 5 = 4$$

'Put your finger on number nine. Count back 5.'

Year 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.



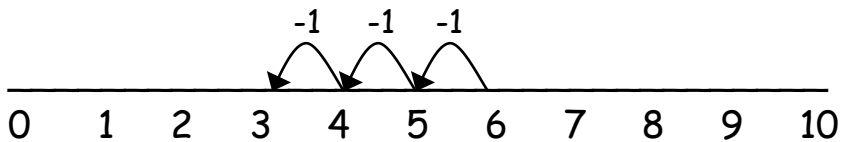
They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line.

Role play, change from 20p coin in shop.

Counting back using an object as a counter.

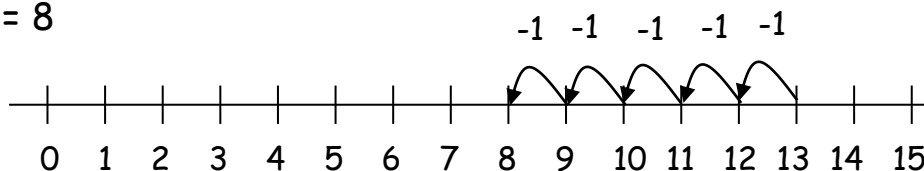
Counting back in tens. Starting on multiple of 10 (e.g. 40, 30, 20, 10), then starting on any number.

$$6 - 3 = 3$$



Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones. They also may use their fingers.

$$13 - 5 = 8$$



Add and subtract one-digit and two-digit numbers to 20, including zero

Missing number problems e.g. $20 - \square = 9$ $15 - 9 = \square$ $\square - \square = 11$

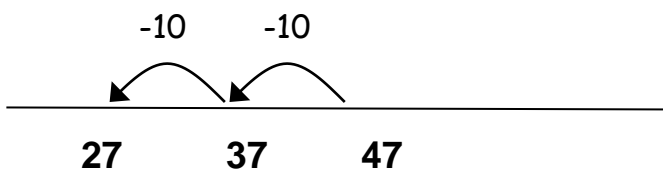
Stage 2 (approx Y2)

Children will begin to use empty number lines to support calculations.

Counting back

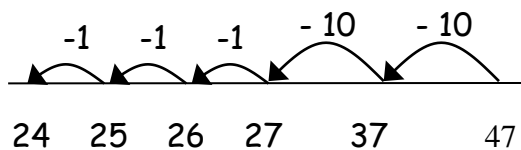
- ✓ Count back in multiples of 10.
- ✓ Count back in multiples of 10 on a number line.

$$47 - 20 =$$



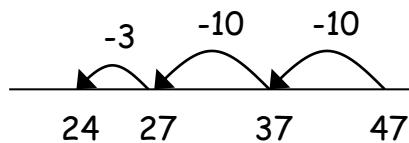
- ✓ First count back in tens then in ones.

$$47 - 23 = 24$$



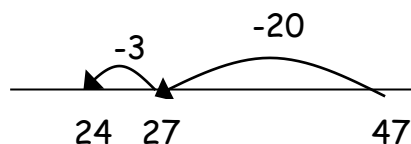
- ✓ Help children to become more efficient by subtracting the ones in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



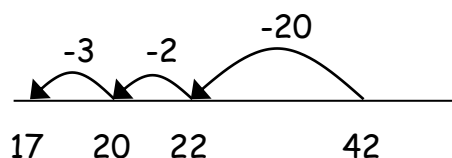
- ✓ Subtract the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



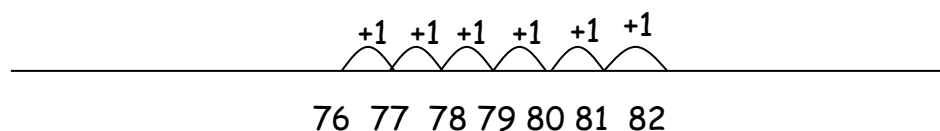
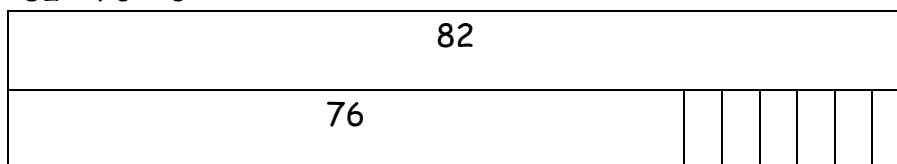
Counting on

Only if the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.



Count on from 76 to 82.

$$82 - 76 = 6$$



Help children to become more efficient with counting on by:

- ✓ Subtracting the ones in one jump;
- ✓ Subtracting the tens in one jump and the ones in one jump;
- ✓ Bridging through ten.

Subtract larger numbers using concrete objects, pictorial representations, and mentally, including:

- ✓ a two-digit number take away units
- ✓ a two-digit number take away tens
- ✓ take away a two-digit number from another 2 digit number

Show that subtraction of one number from another number cannot be done in any order. Check answers using the inverse (addition).

Using a vertical method

$$54 - 22 = 32$$

After practically using the Dienes blocks, children can draw the blocks to help them to solve subtraction calculations. They will then record the calculation vertically imagining the Dienes blocks.



50	4	
— 20	2	
<hr/>		
30	2	= 32
<hr/>		

Stage 3 (approx Y3)

Children will continue to use empty number lines with increasingly large numbers.
Children will continue to use informal pencil and paper methods (jottings, bar models, diagrams) to support, record and explain partial mental methods building on existing mental strategies.

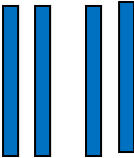


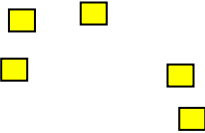
Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning and Dienes materials to show the decomposition of the number.

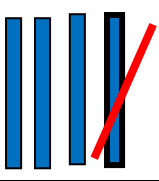
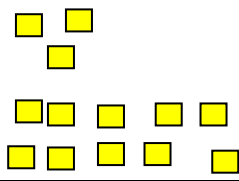
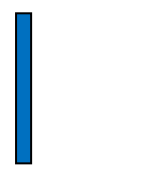
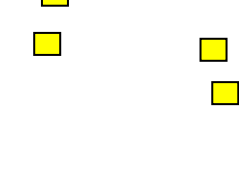
Initially, the children will be taught using examples that do not need the children to exchange.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 \\ 50 \\ \hline 30 \end{array} \quad \begin{array}{r} 9 \\ 7 \\ \hline 2 \end{array} = 32$$

Use Dienes and draw diagrams

Hundreds	Tens	Units	
			$\begin{array}{r} 43 \\ - 15 \\ \hline \end{array}$
			

Start with the units column. Can I take 5 away from 3 easily? I need to exchange one of my tens for ten units.

Hundreds	Tens	Units
		
		

From this the children will begin to record the exchanging process

$$\begin{array}{r} 43 \\ - 15 \\ \hline \end{array}$$

Step 1

$$\begin{array}{r} 40 \quad 3 \\ - 10 \quad 5 \\ \hline \end{array}$$

The calculation should be read as e.g. take 5 from 3.

Step 2

$$\begin{array}{r} 30 \quad 13 \\ - 10 \quad 5 \\ \hline 20 \quad 8 \end{array} = 28$$

Now we can take 5 from 13.

This would be recorded by the children as:

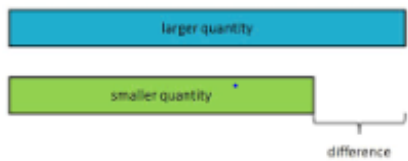
$$\begin{array}{r} 30 \\ \cancel{40} \text{ and } 13 \\ - 10 \text{ and } 5 \\ \hline 20 \text{ and } 8 = 28 \end{array}$$

This then becomes:

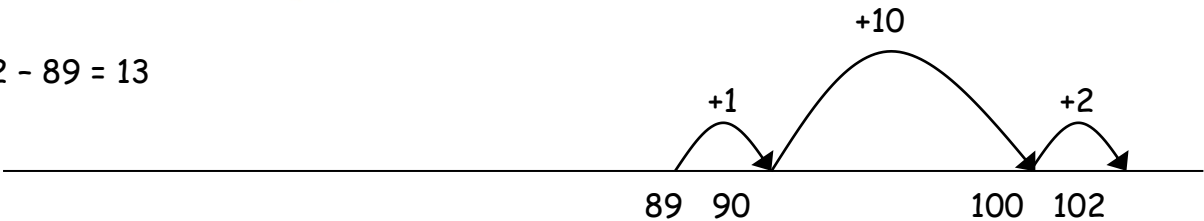
$$\begin{array}{r} 3 \quad 13 \\ \cancel{4} \quad \cancel{3} \\ - 1 \quad 5 \\ \hline 2 \quad 8 \end{array}$$

Children should know that ones line up under ones, tens under tens, and so on.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.
 Use bar modelling to reinforce the concept.



102 - 89 = 13



Stage 4 (approx Y4)

Partitioning and decomposition

Continue to use Dienes and draw diagrams for more complex questions.

Hundreds	Tens	Units

$$\begin{array}{r} 243 \\ -115 \\ \hline \end{array}$$

Hundreds	Tens	Units

$$\begin{array}{r} 313 \\ 243 \\ -115 \\ \hline 127 \end{array}$$

Decomposition *Concise Stage

$$\begin{array}{r} 614 \\ \cancel{75}^{14} \\ - 86 \\ \hline 668 \end{array}$$

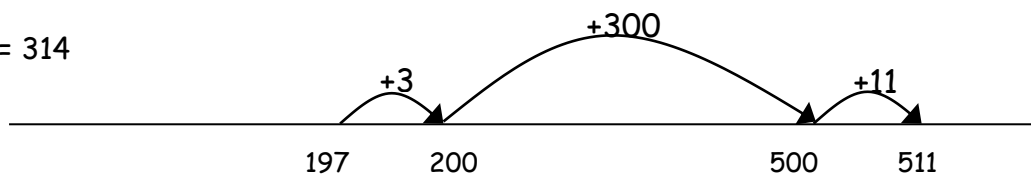
Children should:

- ✓ move onto subtractions with exchanges in two columns.
- ✓ be able to subtract numbers with different numbers of digits;
- ✓ using this method, children should also begin to find the difference between two three-digit sums of money, with 'adjustment' from the pence to the pounds;
- ✓ know that decimal points should line up under each other (especially with money and measures)
- ✓ Move onto exchanging across 0.

NB If your child has reached the **CONCISE STAGE***, he/she will then continue this method through into years 5 and 6. They should not need to go back to using the expanded methods unless they are experiencing difficulties which can be solved by revisiting the method using Dienes.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line or bar model should still be used.

$$511 - 197 = 314$$



511			
197	3	300	11

Stage 5 (approx Y5)

Decomposition (Concise Stage)

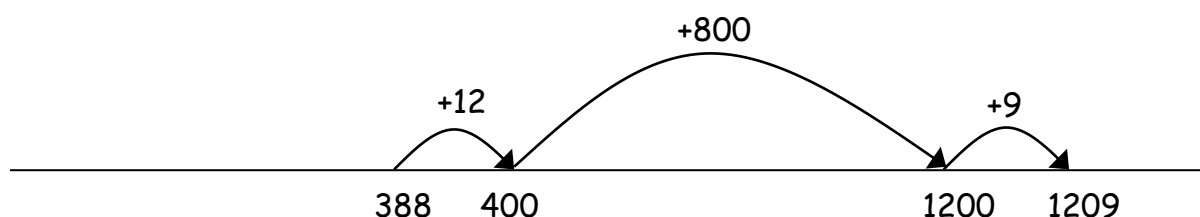
$$\begin{array}{r} 614 \\ \cancel{75}^1 \\ - 286 \\ \hline 468 \end{array}$$

Children should:

- ✓ be able to subtract numbers with different numbers of digits (up to 4 digits);
- ✓ begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other;
- ✓ estimate first and check by using inverse operation.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used. The number lines or bar models should also be used to solve problems involving time, measures and giving change from whole pounds.

$$1209 - 388 = 821$$



£50-£20.99

£50			
£20.99	1p	£20	£9

Stage 6 (approx Y6)

Decomposition

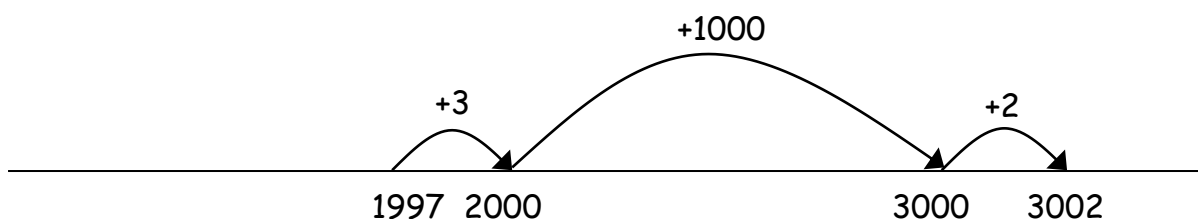
$$\begin{array}{r}
 \overset{5}{6} \overset{13}{4} \overset{1}{6} 7 \\
 - \underline{2684} \\
 3783
 \end{array}$$

Children should:

- ✓ be able to subtract numbers with different numbers of digits:
- ✓ be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places or a different number of decimal places:
- ✓ know that decimal points should line up under each other;
- ✓ solve multi- step problems;
- ✓ continue to use estimation, rounding and inverse checking strategies.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line may be used as aid to mental methods. The number lines or bar models should also be used to solve problems involving time, measures and giving change from whole pounds.

$$3002 - 1997 = 1005$$



The train leaves Edinburgh at 5:22am and arrives at London at 11:16am. How long does the journey take?

11:16am				
5:22am	8mins	30mins	5 hours	16 mins

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to known facts.
e.g. 8×4 is double 4×4

Using multiplication facts

Tables should be taught every day from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day. Parents need to practise these with their children every few days.

Year 1 2 times table 5 times table 10 times table

Year 2 As Year 1 and 3 times tables

Year 3 As above but also;
3 times table 4 times table 8 times table

Year 4 Derive and recall ALL multiplication facts up to 12×12

Years 5 & 6 Derive and recall QUICKLY all multiplication facts up to 12×12 and associated divisions. Once children have mastered these facts, they will be challenged with higher numbers, multiples of 10 and decimals.

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\,000$, $0.3 \times 7 = 2.1$ etc

Use closely related facts already known

$$\begin{aligned} 13 \times 11 &= (13 \times 10) + (13 \times 1) \\ &= 130 + 13 \\ &= 143 \end{aligned}$$

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning

$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 102 \end{aligned}$$

Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

Language of 'whole' and 'part'

How many equal parts make a whole?

In multiplication, we know the parts but do not know the whole.

? Whole		
5 Part	5 Part	5 part

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS

WRITTEN METHODS OF MULTIPLICATION

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

Stage 1 (approx Reception and Y1)

Reception

Counting in twos orally.

Stories e.g. Noah's Ark

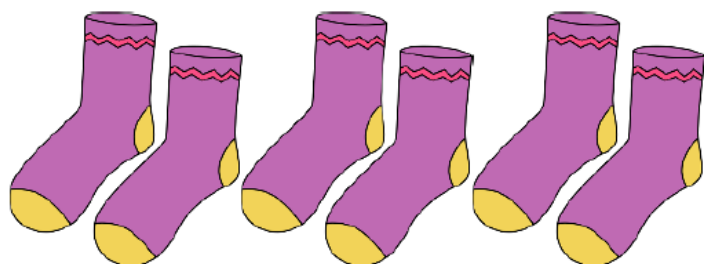
Rhymes such as Sizzling Sausages

Games such as Pairs of socks.

Year 1

Count in 2s, 5s and 10s

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups e.g. How many socks in 3 pairs?



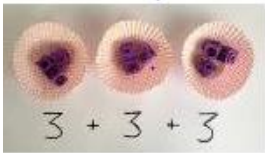
Songs and rhymes

Introduce x symbol

Lots of concrete activities using real objects.

Recording using drawings e.g. drawing butterflies and counting wings.

Using vocabulary such as "cups of", "lots of", "groups of", "multiplied by"



Use different
objects to add
equal groups.

Begin to use arrays with support of the teacher.

$$\begin{array}{ccc} \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \end{array} \quad 2 \times 3 = 6$$

$2 \times 3 = 6$

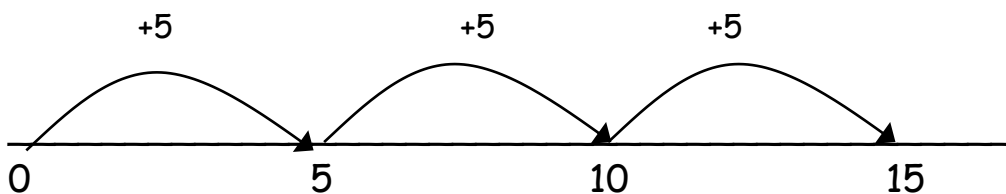
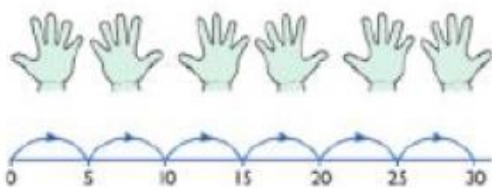
Stage 2 (approx Y2)

Children will develop their understanding of multiplication and use jottings to support calculation during Y2.

Repeated addition

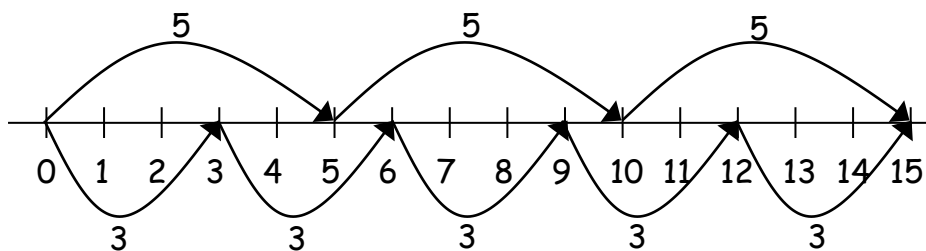
3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

Repeated addition can be shown easily on a number line:



Commutativity

- ✓ Children should know that 3×5 has the same answer as 5×3 . This can be shown on the number line.

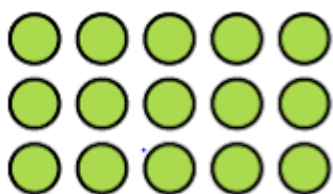


- ✓ This can also be shown through bar modelling using Cuisenaire rods and drawing diagrams.

15				
5	5	5		
3	3	3	3	3

Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$$3 \times 5 = 15$$



$$5 \times 3 = 15$$

Recognise odd and even numbers.

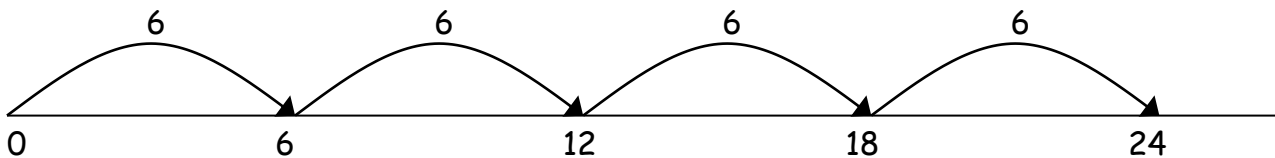
Understand multiplication as scaling (3 times bigger/taller).

Stage 3 (approx Y3)

Children will continue to use:

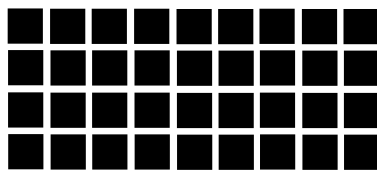
Repeated addition

4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or 4×6 or 6×4
Children should use number lines or bead bars to support their understanding.



Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$$4 \times 9 = 36$$

$$9 \times 4 = 36$$

Scaling

36 is 9 times bigger than 4.
4 is 9 times smaller than 36.

e.g. Find a ribbon that is 4 times as long as the blue ribbon



5 cm



20 cm

✓ Use known facts

e.g. $3 \times 4 = 12$ so $40 \times 3 = 120$ and $3 \times 40 = 120$

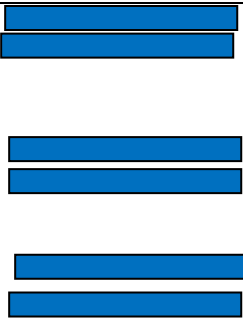
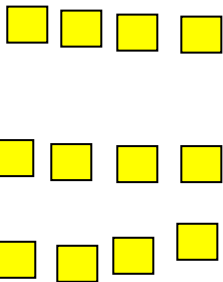
Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \bigcirc = 32$$

- ✓ Use Dienes
 $3 \times 24 =$

	20	4
3		

$$60 + 12 = 72$$

Once children have mastered this with Dienes, they will use record this using the grid method.

X	20	4
3	60	12

Stage 4 (approx Y4)

Grid method

- ✓ Continue with Dienes (see example at end of Year 3)
- ✓ **Practise 3 digit x 1 digit and moving onto 4 digit x 1 digit** (Short multiplication – multiplication by a single digit)

$$146 \times 4$$

Then utilise a grid to find out the exact answer.

X	100	40	6
4	400	160	24

$$\begin{array}{r}
 400 \\
 + 160 \\
 + 24 \\
 \hline
 584
 \end{array}$$

This is nearly 3500 so calculation seems correct.

Column written methods

- ✓ Children who are secure with the grid method of multiplication can move onto the column methods.

$$17 \times 5$$

$$\begin{array}{r} 17 \\ \times 5 \\ \hline 50 \text{ (5} \times 10\text{)} \\ 35 \text{ (5} \times 7\text{)} \\ \hline 85 \end{array}$$

- ✓ This then becomes:

$$17 \times 5$$

$$\begin{array}{r} 17 \\ \times 5 \\ \hline 85 \\ 3 \end{array}$$

Stage 5 (approx Y5)

4 digit x 1 digit

Starting with grid method and then moving onto compact column written method.

Compact column written method:

$$1487 \times 5$$

Children will **approximate** first in order to check the size of their calculation:

e.g. 1487×5 is approximately $1500 \times 5 = 7500$

$$\begin{array}{r} 1487 \\ \times 5 \\ \hline 7435 \\ 2 \text{ } 43 \end{array}$$

2 digit x 2 digit and moving onto 3 digit x 2 digit and then 4 digit x 2 digit. (Long multiplication – multiplication by more than a single digit)

- ✓ Early stage to show which number is being partitioned.

$$72 \times 12 = 864$$

$$\begin{array}{r} 72 \\ \times 12 \\ \hline 144 \quad (2 \times 72) \\ 720 \quad (10 \times 72) \\ \hline 864 \end{array}$$

- ✓ Long multiplication

$$\begin{array}{r} 72 \\ \times 12 \\ \hline 144 \\ 720 \\ \hline 864 \end{array}$$

Children can cross out the units once they have multiplied with them.

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

Children will approximate first in order to check the size of their calculation.

$$\begin{array}{r} 4.92 \\ \times 3 \\ \hline 14.76 \\ 2 \end{array}$$

Which is nearly 15 so calculation seems correct

Stage 6 (approx Y6)

Those who find the column method difficult for short multiplication can continue to use the grid method.

Long Multiplication

3 digit x 2 digit moving onto 4 digit x 2 digit (Long multiplication – multiplication by more than a single digit)

$$487 \times 15$$

Children will approximate first in order to check the size of their calculation:

eg 487×15 is approximately $500 \times 15 = 7500$

$$\begin{array}{r}
 {}^44{}^38\ 7 \\
 \times \quad \underline{1\cancel{5}} \\
 2\ 4\ 3\ 5 \\
 4\ 8\ 7\ 0 \\
 \hline
 7\ 3\ 0\ 5 \\
 1\ 1
 \end{array}$$

For children who get confused once they have multiplied the units, they can cross out the units and move onto the tens. Carried digits go at the top except in the final adding part.

Children will continue to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

$$\begin{array}{r}
 {}^24.\ 9\ 2 \\
 \times \quad \underline{1\ 3} \\
 1\ 4.7\ 6 \\
 4\ 9.2\ 0 \\
 \hline
 6\ 3.9\ 6 \\
 1
 \end{array}$$

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

MENTAL CALCULATIONS

These are a **selection** of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2 and doubling is times by 2.

Deriving and recalling division facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day. Parents need to practise these with their children every few days.

Year 2	2 times table	5 times table	10 times table
Year 3	As above but also; 3 times table	4 times table	6 times table
Year 4	Derive and recall division facts for all tables up to 10 x 10		
Year 5 & 6	Derive and recall quickly division facts for all tables up to 12 x 12		

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\,000$, $0.3 \times 7 = 2.1$ etc

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

$378 \div 21$	$378 \div 3 = 126$	$378 \div 21 = 18$
	$126 \div 7 = 18$	

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?

Bar modelling

In division, we know the whole but do not know the parts.

15 Whole		
Part ?	Part ?	Part ?

In division, we know the whole but do not know the number of parts'

27 Whole				
3	3	?	3	3

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

WRITTEN METHODS FOR DIVISION

THE FOLLOWING ARE STANDARDS THAT WE EXPECT THE MAJORITY OF CHILDREN TO ACHIEVE.

Stage 1 (approx Reception and Y1)

Reception

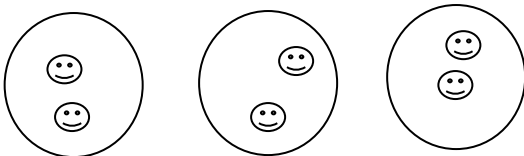
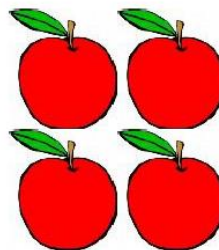
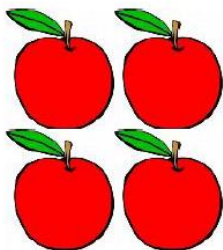
Children will understand 'equal' groups and share items out in play and problem solving.
Sharing out plates, spoons, cups in the home corner.
Role play situations – sharing food out for teddies.
Pouring out drinks from a bottle. How many cups will it fill?

Year 1

They will count in 2s, 5s, 10s and 10s
Children will be taught to share using concrete apparatus,
Folding shapes into 2 equal parts

Sharing equally

Share these 8 apples equally between 2 children. How many will they get each?



Arrays

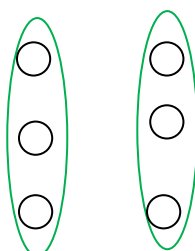
Begin to use arrays with support of the teacher.



$$6 \div 3 = 2$$



$$6 \div 2 = 3$$

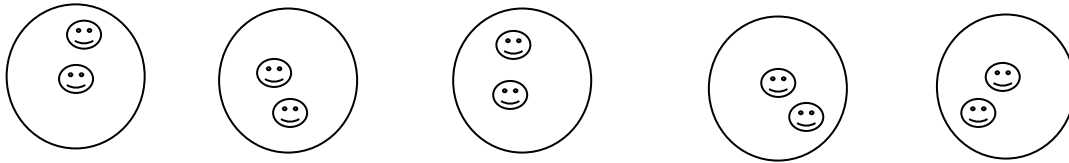


Stage 2 (approx Y2)

Children will develop their understanding of division and use jottings to support calculation

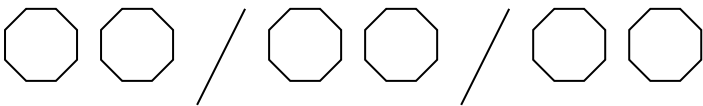
Continue sharing equally

6 sweets shared between 2 people, how many do they each get?



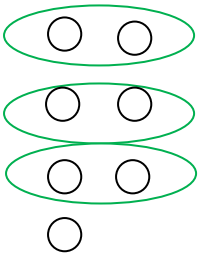
Grouping or repeated subtraction

There are 6 sweets. How many people can have 2 sweets each?



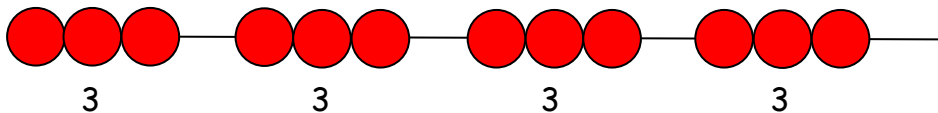
Remainders

There are 7 sweets. How many people can have 2 sweets each?

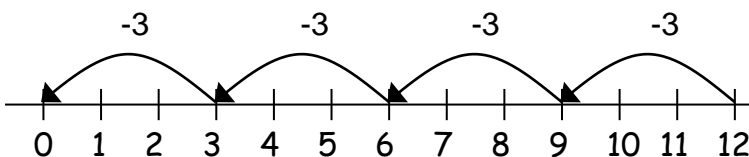


Repeated subtraction using a bead bar or number line

$$12 \div 3 = 4$$



The bead bar will help children with interpreting division calculations such as $12 \div 3$, how many 3s make 12?



Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \div 2 = 4$$

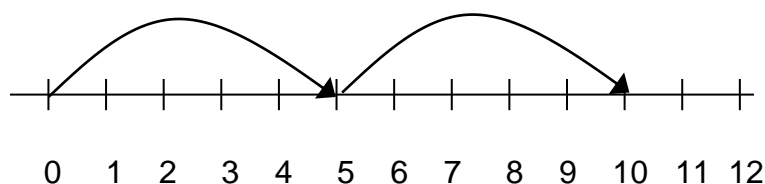
$$20 \div \triangle = 4$$

$$\square \div \triangle = 4$$

Know that division of one number from another cannot be done in any order.

Remainders

$$12 \div 5 = 2 \text{ r } 1$$



Representing division through bar modelling

15		
5	5	5

$$15 \div 3 = 5$$

Stage 3 (approx Y3)

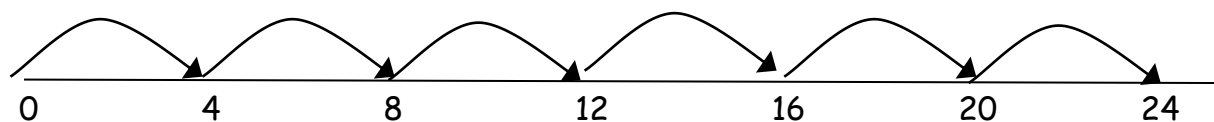
Ensure that the emphasis in Y3 is on grouping (rather than sharing) as numbers will be getting bigger, so not always able to share out.

Children will continue to use:

Children will use an empty number line to support their calculation.

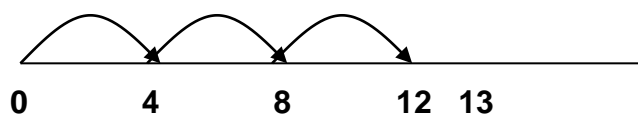
$$24 \div 4 = 6$$

How many lots of 4 are there in 24?



Children should also move onto calculations involving **remainders**.

$$13 \div 4 = 3 \text{ r } 1$$



Children will explore what remainders mean in the context of word problems.

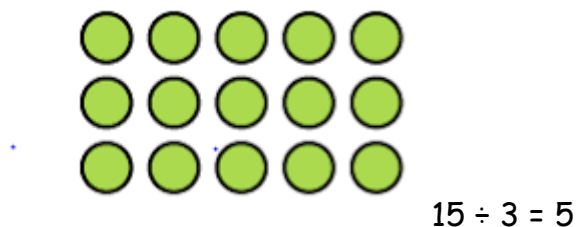
Use symbols to stand for unknown numbers to complete equations using inverse operations

$$26 \div 2 = \square$$

$$24 \div \triangle = 12$$

$$\square \div 10 = 8$$

✓ Use arrays

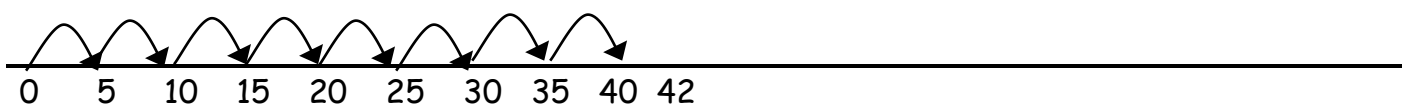


Calculate divisions using the times tables that they know.

Stage 4 (approx Y4)

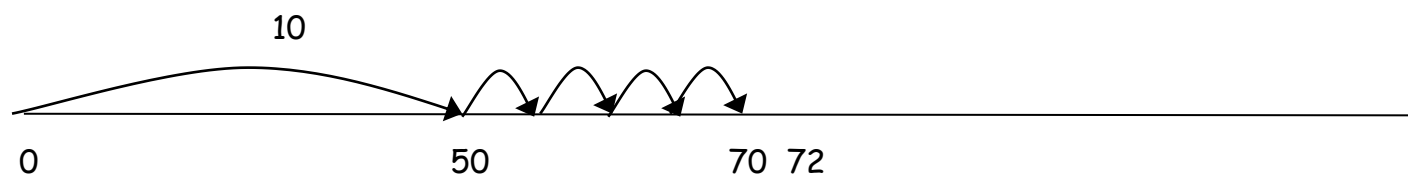
Continue number line subtraction

$$42 \div 5 = 8 \text{ r } 2$$



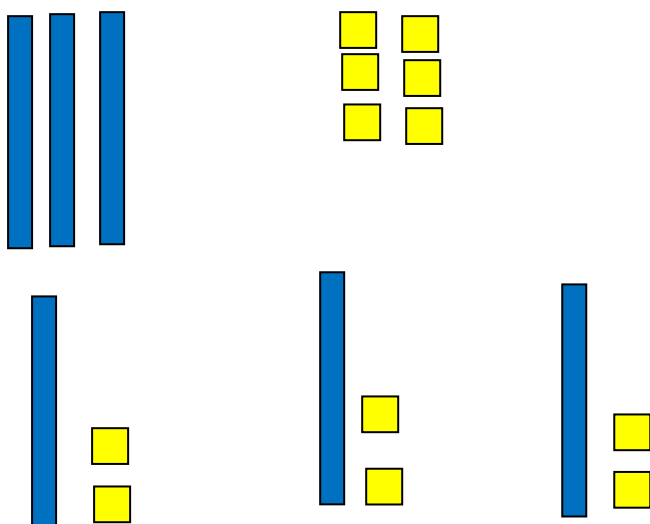
Move onto:

$$72 \div 5 = 14 \text{ r } 2$$

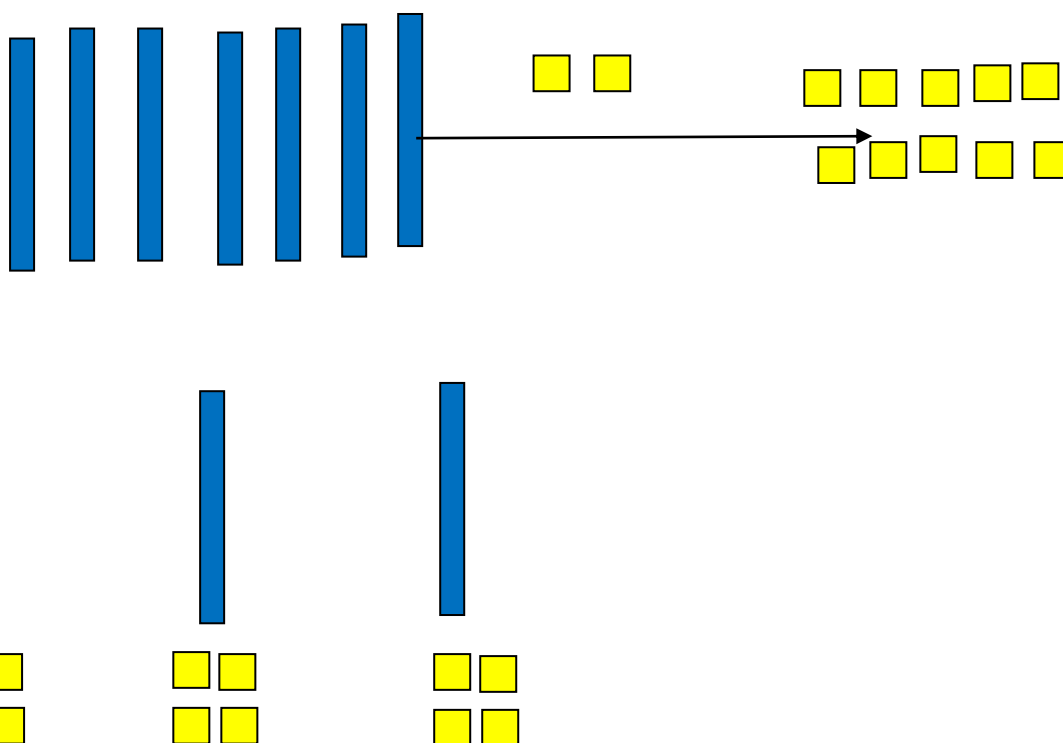


Use Dienes to show bus stop method

$$36 \div 3 =$$



$$72 \div 3 =$$



We can put one ten in each group. We have one ten left over. We exchange this ten for ten units and then share all the units equally among the groups.

Many children will be able to move straight onto bus stop division (without remainders)

$$\begin{array}{r} 12 \\ 7 \overline{) 84} \end{array}$$

Any remainders should be shown as **integers**, i.e. 14 remainder 2 or 14 r 2.

- ✓ *Children need to be able to decide what to do after division and round up or down accordingly.*
- ✓ *They should make sensible decisions about rounding up or down after division. For example $62 \div 8$ is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context*
- ✓ *Use the inverse (multiplication) to check.*

e.g. I have 62p. Sweets are 8p each. How many can I buy? Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed? Answer: 8 (the remaining 6 apples still need to be placed into a box!)

Stage 5 (approx Year 5)

Children will continue to use written methods to solve short division 2 digit \div 1 digit including with remainders.

- ✓ *Extend to 4 digit \div 1 digit*
- ✓ *Extend to decimals with up to two decimal places.*
Children should know that decimal points line up under each other.
- ✓ *Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example $240 \div 52$ is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context. (See Stage 4 examples above)*
- ✓ *Children need to be able to give the remainder as a fraction/decimal, for example, $98 \div 4 = 24 \text{ r } 2 = 24 = 24.5 \approx 25$.*

Stage 6 (approx Year 6)

Children will continue to use written methods to solve short division 2 digit \div 1 digit and 3 digit \div 1 digit.

Long division 3 digits \div 2 digits moving onto 4 digit \div 2 digits

$$432 \div 15 =$$

$$\begin{array}{r} 28 \text{ r}12 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

Answer: 28r 12

Changing the remainder into decimals

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.120} \end{array}$$

- ✓ *Remainders. How to show these depends on the context. Sometimes they will need to be rounded in order to answer a real life problem. Sometimes they should be shown as fractions or decimals, i.e. if the children were dividing 32 by 10, the answer should be shown as $3\frac{2}{10}$ which could then be written as $3\frac{1}{5}$ in its lowest terms or 3.2. Children will continue to round the answers depending on the question.*
- ✓ *Continue to extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.*